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Improving the reliability of *k*-out of-*n* system: The case of series system

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ABSTRACT: This paper addresses the problem of reliability improvement of k -out of - n system. It first looks at how series system is improved through the improvement of individual components with most reliable components. It also used redundancy (cold and hot) to improve the series system reliability.

Keywords: k -out of - n system, Series system, redundancy

Introduction

The prime maintenance objective is to see that a system performs its desired function. The introduction of every new device must be accompanied by provision for maintenance, repair parts, and protection against failure. Where it is necessary to avoid system failure during operation, where such failures are catastrophic and costly, it is imperative to perform planned maintenance actions (preventive maintenance).

It is of great importance to avoid system failure before, during or after operations. The name reliability is given to the field of study that studies the system performance. Failure avoidance before operation is considered as preventive maintenance of the system. A system consists of a number of components. Each component is either in operational or failure state. The status of the system is known when the set of operating components and the set of failed components is specified.

Sometimes components due to age or usage were substituted with the most reliable, new and identical ones. This would make system to operate efficiently and successfully. Thus improving the system reliability.

In reliability engineering, it is a tradition to use redundancy techniques to improve system reliability. A common example of redundancy is the *k-out* of -*n* system in which the system is successful or operational if any of *k* out of *n* components are successful or operational. Example of the *k*- out of- *n* system is the multi-engine system in an airplane.

Reliability has been the major concern for system designers. Redundancy of components is needed to design highly reliable system. Redundancy of components is usually incorporated in a system design for improving and increasing reliability.

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Materials and Method

Reliability is the probability that a system performs its mission successfully. It is the probability that a system will operate satisfactorily for a given period of time.

The *k*- out of- *n* system is an n component system that works if and only if at least *k* of the n components work. Special cases of the *k*- out of- *n* system are parallel and series system. A series system is the one in which all components must be operating for the system to successful or working. It has the representation *n* – out of – *n* .

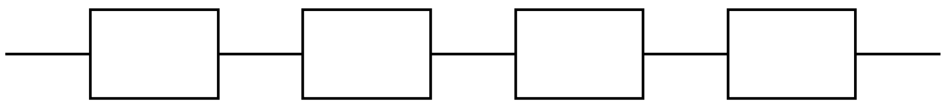

Fig 1. A system with components arranged in series.

A parallel system is an n components system that works if any one component is work. All components must fail before the parallel system fails.

Series system is an n components system that works if and only if all components work.

Result and Discussion

Theorem

The reliability of a series system is given by

$$R = R_1 X\ R_2\ X ... X\ R_n \text{ ………………..(1)}$$

Proof . Let E be the event that the system is working and S_i be the event component *kth* component is working.

Let $R = P(E)$ and $R_i = P(S_i)$, $i = 1, 2, ..., n$. From the probability law of intersection of an independent events,

$$E = S_1 \cap S_2 \cap ... \cap S_n \text{ ………………(2)} \qquad \text{And}$$

$$P(E) = P(S_1)P(S_2)...P(S_n) \text{ ………………………(3)}$$

Hence $R = P(E) = P(S_1)P(S_2)...P(S_n) = R_1 X\ R_2\ X ... X\ R_n$ …………….(4)

Proposition 1:

If the least reliable components are improved in the series system, then the system reliability is improved and $R = R_1^{\ n}$.

Proof:

Let the reliabilities of the components be arranged as

$R_1 > R_2 > R_3 > ... > R_{n-1} > R_n$ …………………(5)

Here R_n is the reliability of least reliable component C_n and R_1 is the reliability of the most reliable component C_1. Let component C_n be improved to the most reliable component C_1, then $R_n = R_1$ and $R = R_1 \, X \, R_2 \, X ... X \, R_n < R_{n-1}$. Thus R_{n-1} is the reliability of least reliable component C_{n-1}.

If $R_1 = R_{n-1}$, then the reliability of the least reliable component C_{n-1} has been improved to the most reliable component C_1.

Continuing in this manner with other least reliable components, we have

$R = R_1 X \, R_2 \, X ... X \, R_n = R_1^{\,n}$. Hence the series system reliability has been improved.

Proposition 2:

Let the random variables $X_1, X_2, ... X_n$ be the lifetime of the series system components. Let another random variables $Y_1, Y_2, ..., Y_n$ be the lifetime of the components which are independent of each other as well as $X_i s$. If only one of them will be used for standby (hot) redundancy in the position of the corresponding components, C_i, respectively. If $X_i s$ are order stochastically as

$X_1 \geq_{st} X_2 \geq_{st} ... \geq_{st} X_n$ And $X_i =_d Y_i$, $i = 1, 2, ..., n$

Then the lifetime of the series system with hot standby redundancy in components C_i

$\Psi_{n/n}\left(X_1, X_2, ..., X_{n-1}, \max\{X_i, Y_i\}, X_{i+1}, ..., X_n\right)$ ……………..(6)

is increasing in $i \in \{1, 2, ..., n\}$ in the sense of ordering stochastic order.

Proposition 3:

Let $X_1, ..., X_n$, $X_1', ..., X_n'$ be independent lifetimes of the components C_i where $X_i =^d X_i'$ for $i = 1, 2, ..., n$ and $X_1 \leq_{st} ... \leq_{st} X_n$. Then the lifetime of the series system $\Psi_{n/n}\left(X_1, ..., X_i \vee X_1', ..., X_n\right)$ is stochastically decreasing in $i \in \{1, 2, ..., n\}$.

Proposition 4:

Let X_i, $i = 1, 2, ..., n$ be mutually independent nonnegative random variables representing the lifetimes of component $C_i s$. Further, let Y be a nonnegative random variable representing the lifetime of a standby redundancy which could be utilized in any position of components. If $X_i s$ are ordered as $X_1 \geq_{lr} ... \geq_{lr} X_n$, then the lifetime of the series system with cold standby redundancy in component C_i $\Psi_{n/n}(X_1, ..., X_{i-1}, X_i + Y, X_{i+1}, ..., X_n)$ is increasing in $i \in \{1, 2, ..., n\}$ in the sense of order stochastic order.

Conclusion

The larger the number of components is, the lower is the reliability of the series arrangement and reliability of a series arrangement is smaller than the reliability of the least reliable component. Thus, $R = R_1 \, X \, R_2 \, X ... X \, R_n < R_k$

In this paper, it is studied ways of improving series system reliability. The analysis is carried out in to different ways: the redundancy improvement and component wise improvement.

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