African Scientist Vol. 11, No. 3, September 30, 2010
Printed in Nigeria
1595-6881/2010 $12.00 + 0.00
© 2010 Klobex Academic Publishers
http://www.klobex.org/afs

AFS 2010092/11307

A replacement problem of n unit parallel system with minimal repair options

Ibrahim Yusuf* and S. M. Umar

Department of Mathematical Sciences, Bayero University, PMB 3011, Kano - Nigeria

(Received June 9, 2010; Accepted July 7, 2010)

ABSTRACT: We consider a replacement problem of n units parallel system, where a component failure is rectified by minimal repairs. We exchanged the system preventively before failure if the number of minimal repairs reaches $m-r,\ldots,m-2,m-1$ and is preventively replace if the sum of minimal repair reaches threshold value m.
Keywords: minimal repairs, parallel system, failure,

Introduction

A parallel system is a system containing n independent identical component and fails whenever all the components failed. Each component is assumed to fail independent of the other components. Such failure is termed independent failures.

Each component failure is removed by minimal repairs and the system is exchanged before all components fail. We allowed $m_1+m_2+\ldots+m_n=m$ minimal repairs to the system. Thus, m_i is allowed to i^{th} components. The system is exchanged preventively before failure after it experienced $m-r,\ldots,m-2,m-1$ number of minimal repairs, and is replaced if the total of minimal repairs reaches a threshold value m.

A cost c_1 is incurred if the system is exchanged before failure and the cost c_2 for replacement is incurred, where $c_1 < c_2$. The objective here is to determine optimal value m^* which minimize the expected cost per unit of time.

---

*To whom correspondence should be addressed> E-mail: ibrahimyusif@yahoo.com

THE NHPP FAILURE

Since components failure are removed by minimal repair, then the random lifetime $\{T_i\}_{i=0}^{\infty}$ where the minimal repair takes place is a non homogeneous Poisson process (NHPP).Let $N(t)$ be the number of minimal repairs done in the interval $[0,t]$, then $\{N(t),\, t \geq 0\}$ is a NHPP with intensity $r(t)$ and mean value function

$$R(t) = \int_0^t r(t)\,dx \qquad (1)$$

MAINTENANCE COST

Since failure occur according to NHPP with mean value $R(t)$, let $T(m)$ be the lifetime of component where m minimal repairs are allowed, then

$$\overline{F}(t) = P\left[T(m) > t\right] = \sum_{I=0}^{m} e^{-R(t)} \frac{\left[R(t)\right]^i}{i!} \qquad (2)$$

The mean time to the mth repair/failure is

$$\sum_{i=1}^{m-1} \int_0^t e^{-R(t)} \frac{\left[R(t)\right]^i}{i!}\,dt \qquad (3)$$

From elementary renewal reward theorem, the expected cost per unit of time for an infinite time span is

$$C(m) \equiv \lim_{t\to\infty} \frac{C'(t)}{t} =$$

$$C(m) = \frac{c_1 \overline{F}(t) + c_2\left[1 - \overline{F}(t)\right]}{\sum_{i=0}^{m-1} \int_0^\infty e^{-R(t)} \frac{\left[R(t)\right]^i}{i!}\,dt} = \qquad (4)$$

Where $\overline{F}(t)$ is define above

$[1]$ have shown that for continuous and increasing $r(t)$, there exist solution that satisfies

$$\sum_{i=0}^{m-1} \int_0^\infty \frac{\left[R(t)\right]^i}{i!} e^{-R(t)}\,dt - (m-1) \geq \frac{c_1}{c_2} \qquad (5) \text{ and}$$

minimize $C(m)$,such solution is m^*.

References

1. Nakagawa, T. Maintenance and Optimum Policy: Handbook of Reliability Engineering: Springer. 2003, 372-373
2. Boland,P.J. and Proschan, F. Stochastic Order in System Reliability Theory:
3. Nakagawa, T. Kowada M. Analysis of a system with minimal repair and its application to replacement policy. Eur J Oper Res 1983: 12, 176-82
4. Nakagawa, T. Optimal number of failures before replacement time. IEEE Trans Reliab 1983; R-32: 115-6
5. Aven T. Optimal replacement under a minimal repair strategy- a general failure model. Adv Appl Prob 1983: 15; 198-211